

**Solved Examples 8: Availability**

**Example 8.1** In Example 7. 1, calculate the availability loss as well as Second Law efficiency.

**Answer:**

Availability paid by the hot heat reservoir is  $|Q_{res}| (1 - T_{amb}/T_{res}) = 240 \text{ J}$

Availability gain of air is zero since it is at ambient temperature.

Hence total availability loss is 240 J and Second Law efficiency is  $\eta_{2nd} = 0$

**Example 8.2** In Example 7. 2, assuming that the cycle was performed on an ideal gas in open system (mass flow rate is 1kg/s), and that heat was added from a reservoir at 300°C and rejected to ambient air at 0°C, find the availability loss as well as Second Law efficiency of each process

**Answer:**

- **Process 1-2: Reversible isobaric heating**

Availability paid by the hot heat reservoir is  $A_{paid} = |Q_{res}| (1 - T_{amb}/T_{res})$

$$|Q_{res}| = \dot{m}_{air} (h_2 - h_1) = 221.1 \text{ W}; T_{amb} = 273, T_{res} = 573 \Rightarrow A_{paid} = 115.76 \text{ W}$$

Availability gained by air is  $A_{gained} = \dot{m}_{air} [(h_2 - h_1) - T_{amb} (s_2 - s_1)] = 81.92 \text{ W}$

Availability loss  $A_{loss} = A_{paid} - A_{gained} = 33.84 \text{ W}$

N.B. You can easily verify that  $A_{loss}$  is also  $T_{amb} \Delta S_{universe} = T_{amb} (\Delta S_{res} + \Delta S_{air})$

$$\text{where } \Delta S_{res} = Q_{res} / T_{res}$$

Second Law efficiency is  $\eta_{2nd} = A_{gained} / A_{paid} = 70.77 \%$

- **Process 2-3: Irreversible expansion**

Availability paid by air is  $A_{paid} = \dot{m}_{air} [(h_2 - h_3) - T_{amb} (s_2 - s_3)] = 303.495 \text{ W}$

Availability gained is work  $A_{gained} = h_2 - h_3 = 276.878 \text{ W}$

Availability loss  $A_{loss} = A_{paid} - A_{gained} = 26.6175 \text{ W}$

(N.B. you can easily verify that  $A_{loss}$  is also  $T_{amb} \Delta S_{universe} = T_{amb} (\Delta S_{air})$ )

Second Law efficiency is  $\eta_{2nd} = A_{gained} / A_{paid} = 91.297 \%$

- **Process 3-4: Reversible isothermal cooling**

Availability paid by air is  $A_{paid} = \dot{m}_{air} [(h_3 - h_4) - T_{amb} (s_3 - s_4)] = 4.3246 \text{ W}$

Availability gained by cold reservoir is  $A_{gained} = |Q_{res}| (1 - T_{amb}/T_{res}) = 0 \text{ W}$

Availability loss  $A_{loss} = A_{paid} = 4.3246 \text{ W}$

(N.B. you can easily verify that  $A_{loss}$  is also  $T_{amb} \Delta S_{universe} = T_{amb} (\Delta S_{air})$ )

Second Law efficiency is  $\eta_{2nd} = A_{gained} / A_{paid} = 0 \%$

- **Process 4-1: Reversible adiabatic compression**

This is a reversible adiabatic process, hence:

Availability loss  $A_{loss} = 0 \text{ W}$

Second Law efficiency is  $\eta_{2nd} = 100 \%$

- **It appears from cycle availability analysis that the worse process (highest availability loss) requiring to be improved is process 1-2**

**Example 8.3** In example Example 7. 7, assuming ambient conditions are 27°C and 1 bar, find the rate of availability loss as well as Second Law efficiency

**Answer:**

At ambient conditions:  $h_0 = 113.18 \text{ kJ/kg}; s_0 = 0.3946 \text{ kJ/kg K.}$

Knowing that availability of a flowing fluid due to its enthalpy is  $A = \dot{m} [h - h_0 - T_0(s - s_0)]$

Availability paid =  $A_{s,in} + A_{w,in} = 308 \text{ kW}$

Availability gained =  $A_{s,out} = 302.08 \text{ kW}$

Rate of availability loss is  $\dot{I} = T_{amb} dS_{irrev}/dt = (273+27) * 0.0229 = 5.93 \text{ kW}$

Second Law efficiency is  $\eta_{2nd} = A_{gained} / A_{paid} = 98 \%$

**Example 8.4** A rigid tank of volume  $0.2 \text{ m}^3$  initially contains  $0.652 \text{ kg}$  of  $\text{H}_2\text{O}$  at  $2 \text{ bar}$ . A heat reservoir at  $500^\circ\text{C}$  is used to heat the tank until the pressure inside it reaches  $10 \text{ bar}$ . Find initial and final temperatures inside the tank, heat and work exchanged by  $\text{H}_2\text{O}$ , entropy change of the universe as well as availability change due to this process (ambient conditions  $27^\circ\text{C}$ ,  $1 \text{ bar}$ ) and Second Law efficiency. Is it reversible?

**Answer**

*System:* 3 systems: 1) Water (all phases) in the tank (closed); 2) The heat reservoir; 3) Universe (sum of all systems)

*Conservation:* Only energy

*Energies:* Heat, internal energy. No Work.

*Process:* Isochoric

*Properties:* The tank: Initial:  $m, V (\Rightarrow \text{get } v_1), P_1 \Rightarrow \text{complete}$ ; Final:  $P_2, v_2 = v_1 \Rightarrow \text{complete}$ ; The Reservoir:  $T_{res}$  given

*Model:* Steam tables

*Extensive:* yes, 2 values:  $m V$  in tank  $\Rightarrow v$

**Ambient conditions:**

At ambient conditions:  $u_0 = 113.176 \text{ kJ/kg}$ ;  $s_0 = 0.3946 \text{ kJ/kg K}$ ,  $v_0 = 0.001004 \text{ m}^3/\text{kg}$

**System 1: Tank**

- **Initial state:**

specific volume  $v_1 = V / m = 0.2/0.652 = 0.306748 \text{ m}^3/\text{kg}$ , pressure  $P_1 = 2 \text{ bar}$ .

From saturation steam tables at  $2 \text{ bar}$ :  $v_{f1} = 0.0010605$ ,  $v_{g1} = 0.8857$

For wet steam;  $x_1 = (0.306748 - 0.0010605) / (0.8857 - 0.0010605) = 0.346$

$u_{f1} = 504.49$ ;  $u_{g1} = 2529.5$ ;  $\Rightarrow u_1 = u_{f1} + x_1 (u_{g1} - u_{f1}) = 1204.2 \text{ kJ / kg}$

$s_{f1} = 1.5301$ ;  $s_{g1} = 7.1271$ ;  $\Rightarrow s_1 = s_{f1} + x_1 (s_{g1} - s_{f1}) = 3.4641 \text{ kJ / kg K}$

availability  $\phi_1 = (u_1 - u_0) - T_0(s_1 - s_0) + P_0(v_1 - v_0) = 200.75 \text{ kJ/kg}$

- **Final state:**

Given:  $v_2 = v_1 = 0.306748$ ;  $P_2 = 10 \text{ bar}$

From saturation steam tables at  $10 \text{ bar}$ :  $v_{g2} = 0.1944 < v_2 \Rightarrow \text{superheated}$

From superheat tables at  $P_2, v_2$ :  $T_2 = 400^\circ\text{C}$

$u_2$  from steam tables at  $10 \text{ bar}, 400^\circ\text{C}$ :  $2957.3 \text{ kJ/kg}$

$s_2$  from steam tables at  $10 \text{ bar}, 400^\circ\text{C}$ :  $7.4651 \text{ kJ/kg K}$

availability  $\phi_2 = (u_2 - u_0) - T_0(s_2 - s_0) + P_0(v_2 - v_0) = 754.78 \text{ kJ/kg}$

$\Rightarrow Q_{tank} = m (u_2 - u_1) = 1143 \text{ kJ}$

**System 2: Heat Reservoir**

For reservoir  $\Delta S_{res} = Q_{res} / T_{res} = -1143 / (500+273) = -1.4787 \text{ kJ/K}$

**System 3: Universe**

$\Delta S$  for Universe =  $m(s_2 - s_1) - 1.4787 = 1.1272 > 0 \Rightarrow \text{irreversible}$

Availability paid by the hot heat reservoir is  $A_{paid} = |Q_{res}| (1 - T_{amb}/T_{res}) = 699.4 \text{ kJ}$

Availability gained by steam is  $A_{gained} = m (\phi_2 - \phi_1) = 361.227 \text{ kJ}$

Availability loss  $A_{loss} = A_{paid} - A_{gained} = 338.17 \text{ kJ}$

N.B. You can easily verify that  $A_{loss}$  is also  $T_{amb} \Delta S_{universe}$

Second Law efficiency is  $\eta_{2nd} = A_{gained} / A_{paid} = 51.65 \%$